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## STATISTICAL METHOD IN FORESTRY

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The profession of forestry in the United States is dedicated to the development and sustained use of forestlands—comprising about one-third of our land area—and their resources of forest, forage, soil and water. Management of forest properties of the order of tens, or hundreds of thousands of acres generates problems involving inventory and growth of standing timber, utilization of forest products, control of fire, soil erosion and the depredations of disease and insects, methods of harvesting timber so as to insure natural reproduction or the conservation of water and afforestation.

The solution of most of these implies application of the statistical method; not always, perhaps, in such refined form as to gratify the professional statistician, but usually with sufficient effectiveness for the particular purpose.

One of the common activities of the practicing forester is the taking of a sample inventory or "cruise" of standing timber before sale or purchase. In this work he has, for generations indeed, been making use of the method of double sampling—a statistical technique commonly believed to be very modern. The forester has found it time-consuming and expensive to measure the volume of all the individual trees, say in board feet or in cords, in his sample of plots within the area under consideration. In consequence, he may merely tally the frequency distribution of tree diameters on all the sample plots, a comparatively

quick operation for the number of trees involved; while the average tree volume in each diameter class, derived from a free-hand curve of volume on diameter, is based upon an independent sample of volume measurements on 50 or more individual sample trees.

Only an insignificant proportion of such routine sampling jobs provides for randomization and the reckoning of sampling error. The practicing forester does not take readily to randomization of sampling in the field work of the forest inventory; for the timber cruise is practically always a joint project with forest mapping—of cover-type areas at least, and often of age-class areas and topography as well—for which random sampling is wholly inadequate. Furthermore, in the administration of some thousands of acres, even a very precise estimate of the total volume of standing timber thereon is not in itself sufficiently useful information. An additional requirement is that its distribution be given according to species groups within fairly small subdivisions (such as 40-acre compartments) of the property.

Systematic sampling conforms with these needs of forest management. Tree measurements are taken on parallel strips, usually one chain wide, or on lines of sample plots. They cover, perhaps, 1 to 5 percent of the area if the population sampled extends over several thousand acres, and 10 to 20 percent if over several hundred.

Although representative random sampling of the forest inventory has been amply demonstrated (17), the practicing forester retains the conviction, based on long experience, that systematic sampling is more representative than representative random sampling as the latter term is understood by the statistician. This predisposition to systematic sampling led Osborne (15) to study the comparative precision of systematic and random sampling of cover-type areas, and Hasel (9) to an analogous study of timber-volume samplings. Each concluded that the forester's persuasion is well-founded in fact.

Their findings have gratified those foresters of the U. S. Forest Service, charged with the responsibility for the nation-wide forest survey, initiated about 1930. In the Lake States, South Atlantic, and Gulf States the forest survey had relied upon field sampling of a systematic pattern in which each "release unit" of several million acres was traversed with a system of parallel lines, 10 miles apart. At 10-chain distances along the lines, the required data were taken, usually on fifth-acre sample plots. There have been no serious complaints regarding either the cost or the reliability of the survey estimates of forest areas, volume or growth of standing timber, according to the more important cover types.

When required to sample less familiar populations than areas of standing timber, the forester raises little or no objection to the use of random sampling. Thus the erosion control survey of the Tennessee River watershed (19, 20) consisted of data taken according to a representative random sampling design. The watershed of 26 million acres was divided into 324 blocks of equal area, and a random sample of two 1,100-acre plots was drawn from each block. The sampling scheme was practical and efficient because excellent planimetric maps showing forested and non-forested areas had already been completed by the engineers of the Tennessee Valley Authority. Coverage was about 2.7 percent, and the standard errors about 8 percent, at a cost of only \$4,300 for the field work.

Another illustration is afforded by the annual sample census of U. S. lumber production by the Bureau of the Census and the

Forest Service. In the twelve western states no sampling is involved since the population of sawmills—relatively few and mostly large—is completely canvassed. But in the remaining States, some 36 thousand sawmills were reported active in 1942. As their production varied greatly, the population of mills in each state was divided into five production classes in thousands of board feet as follows: 1-49, represented by 10,189 mills; 50-499, by 16,125 mills; 500-999, by 4,560 mills; 1,000-4,999, by 4,756 mills; 5,000+, by 538 mills. Since the standard deviation of production among mills within classes varied widely from one production to another, sampling was not proportional to the number of mills. Instead the sample for each production class consisted of 2, 14, 22, 66 and 100 percent, respectively, of the number of sawmills therein.

Investigations on the effects of grazing by livestock upon forage production and utilization in the western National Forests depend upon accurate sampling appraisals. Forage varies considerably in the weight of plant material produced by each species in a highly variable population. Complete reliance upon direct sampling of weight on sufficient numbers of clipped plots is too laborious and expensive. With practice, on the other hand, weight of forage can be estimated rapidly and fairly accurately by eye. Hence the clue to efficient sampling of forage values lies in adjusting eye-estimates of weight to weight by a scale-balance by double sampling (21). The regression equation of measured weight on the corresponding eye-estimate is derived from the data of relatively few plots for which both values are taken. Upon inserting into the equation the average eye-estimate of weight over the aggregate of all plots of a given observer, the adjusted average weight is readily calculated.

Practical methods for sampling populations of depredating forest insects, and of fur and game animals, have not been completely achieved although progress has been shown (7, 16).

The statistical method is indispensable in experimental forestry. Ever since the establishment of the eleven regional forest and range experiment stations about 1926, programs



of in-service training in statistical method have been provided by the Forest Service and by the Graduate School of the Department of Agriculture. Perhaps the most notable of these was the course of lectures at Asheville, N. C., by Professor R. A. Fisher in 1936 on the design of experiments, attended by a group of forty foresters from the experiment stations and forestry schools. Fisher's appetite for practical problems, his ready comprehension of intricate detail, and his helpful advice on modes of solution were a stimulating experience to men somewhat accustomed to consider research in forestry as something unique.

Experiments in cutting for sustained timber yield, or for maximum water yield and control of erosion in watershed management, have been successfully and efficiently pursued through replication in randomized blocks (14). Treatments are commonly the method of over-story cutting, of grazing, of ground preparation, or the use of fire as a silvicultural tool. Imposed at two or three levels, they fit well into factorial designs. A singularity of this class of problem is the large plot-size of five acres or so, such that a single block may cover 40 acres or more. They lend themselves well, however, to the study of subsidiary effects—such as may be ascribable to methods of brush disposal—by split-plot arrangements.

Sustained maximum timber yield is the objective of annual or periodic harvesting of mature trees whether singly or in groups. The residual stand must be equitably distributed among the younger age classes—including the new natural reproduction—for best development. Successful natural reproduction is the test of silviculture. It depends, in the first place, upon the adequacy and dispersal of the seed supply in both harvested and residual timber; and in the second place, upon the suitability of the forest floor after harvesting or treatment, as a seedbed for germination and growth of seedlings. The factors of seed supply and its dispersal in natural reproduction are estimated quantitatively by sampling with seed-traps (10). The net result of supply, dispersion and establishment can be investigated through regression analysis (12) in which the dependent variate, the proportion of mil-acre quadrats stocked with one or more seedlings, is transformed to probits (1) and

plotted against the average logarithm of the number of seedlings to the quadrat among the plots of the timber-cutting experiment.

Standard designs in randomized blocks or Latin squares are used regularly in experiments on artificial reproduction. These include tests of the effects of fertilizer or other soil treatments on the quality of nursery stock, or of the quality of nursery stock on the survival rate and early growth of plantations. Studies in afforestation, however, can become unusually complex. Certain classes of "treatment" cannot be imposed at will on a given experimental area, but require preparatory exploration for potential block situations where they already prevail. Confounding of certain effects is the inevitable consequence. Thus in his search for practical forestation methods of converting low-value Ozark oak forests to mixtures of oak and shortleaf pine, Chapman (4) was forced to confound two very pertinent factors of environment—namely, density of oak overwood and quality of site—with the blocks of his experiment. In this case however, the loss of useful information was trivial since the chief interest lay not in the main effects of these factors, but in their more precisely evaluated interactions with treatments imposed within the blocks; such as the interaction of density of overwood with root-pruning, and age, of the planted stock.

The need for efficient statistical tools in forestry had first been felt in correlation work involving the graphic expression of timber-tree volume in terms of tree-size, and the growth of even-aged stands. Such relationships are invariably curvilinear and they are usually solved by graphical analysis (13). Theoretical regression equations and appropriate weights for observational data are still in the process of development. In the meantime, however, a graphic control of free-hand curves of a dependent variate on each of two or more independent variates is provided by the alignment chart as developed by Bruce and Reineke (2). Initially the relation among the variates is approximated graphically or by a linear regression equation. Through successive adjustment of its scales the chart is transformed to the final graphic expression of the relationship in question. Even when the form of regression equation is acceptable theoretically

the alinement chart serves as an excellent graphic test of linearity (3 Sec. 151).

Investigations dealing with forest mensuration now consistently make use of regression analysis. Up to about fifteen years ago emphasis was placed on yield studies (growth curves) of even-aged stands of the more important timber types. Recent emphasis has been on quantitative definition of degree of stocking (5, 6) and the rate at which full stocking of timber stands is approached; to sample scaling (11) and the analysis of lumber volume according to tree species and lumber grade as manufactured from sawlogs of various sizes (18). Regression analysis has also been useful in elucidating the effect of fire on gum yield (for turpentine, rosin, etc.) of

longleaf and slash pine (8); and in describing the effect of rainfall distribution on the growth of timber trees and forage grasses.

While practitioners are the clientele of the research forester, they are busy and practical men and have little interest in the statistical method as such. It therefore devolves upon the research forester to present the results of investigation or experiment in simple and attractive terms so that their import can be grasped readily. Hence the statistical method is seldom mentioned by the authors—and not always recognized by the readers—of the 10 or 15 percent of the papers which are based upon its application among those published in the professional *Journal of Forestry*. To the practicing forester this is as it should be.

- (1) Bliss, C. I. The calculation of the dosage-mortality curve. *Ann. App. Bio.* 22:134-167. 1935.
- (2) Bruce, D., and L. H. Reineke. Correlation alinement charts in forest research. U.S.D.A. Tech. Bull. No. 210. 1931.
- (3) Bruce, D. and F. X. Schumacher. Forest mensuration. McGraw-Hill Book Co. 2nd ed. 425 pp. 1942.
- (4) Chapman, A. G. Classes of shortleaf pine nursery stock for planting in the Missouri Ozarks. *Jour. For.* 42:818-826. 1944.
- (5) Chisman, H. H. and F. X. Schumacher. On the tree-area ratio and certain of its applications. *Jour. For.* 38:311-317. 1940.
- (6) Gevorkiantz, S. R. Measuring stand normality. *Jour. For.* 42:503-508. 1944.
- (7) Green, R. G. and C. A. Evans. Studies on a population cycle snowshoe hares on the Lake Alexander area. I Gross annual censuses, 1932-1939.
- (8) Harper, V. L. Effects of fire on gum yields of longleaf and slash pines. U.S.D.A. Circular 710. 1944.
- (9) Hasel, A. A. Sampling error in timber surveys. *Jour. Agri. Res.* 57:713-736. 1938.
- (10) Jemison, G. M. and C. F. Korstian. Loblolly pine seed production and dispersal. *Jour. For.* 42:734-741. 1944.
- (11) Lexen, Bert. Sale of stumpage on the basis of tree measurement. *Jour. For.* 40:845-853. 1942.
- (12) Lynch, D. W. and F. X. Schumacher. Concerning the dispersion of natural reproduction. *Jour. For.* 39:49-51. 1941.
- (13) Meyer, W. H. Yield of even-aged stands of ponderosa pine. U.S.D.A. Tech. Bull. No. 630. 1938.
- (14) Niederhof, C. H. and H. G. Wilm. Effect of cutting mature lodgepole-pine stands on rainfall interception. *Jour. For.* 41:57-61. 1943.
- (15) Osborne, J. G. Sampling errors of systematic and random surveys of cover-type areas. *Jour. Am. Stat. Assn.* 37:256-264. 1942.
- (16) Prebble, M. L. Sampling methods in population studies of the European spruce sawfly in eastern Canada. *Transactions of the Royal Society of Canada. Section V.* 37:93-126. 1943.
- (17) Schumacher, F. X. and R. A. Chapman. Sampling methods in forestry and range management. Duke Univ. School of For. Bull. 7. 1942.
- (18) Schumacher, F. X. and H. E. Young. Empirical log rules according to species groups and lumber grades. *Jour. For.* 41:511-518. 1943.
- (19) Selgworth, K. J. and J. E. Snyder. Further notes on work unit erosion control surveys. *Jour. For.* 39:693-695. 1941.
- (20) Selgworth, K. J. and J. E. Snyder. The erosion control job in the Tennessee Watershed. *Jour. For.* 41:442-443. 1943.
- (21) Wilm, H. G., D. F. Costello and G. E. Klipple. Estimating forage yield by the double-sampling method. *Jour. Am. Soc. Agron.* 36:194-203. 1944.

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# TEACHING STATISTICS AT THE DEPARTMENT OF AGRICULTURE GRADUATE SCHOOL IN WASHINGTON

The Graduate School in the Department of Agriculture occupies a unique position in education. The faculty is composed almost entirely of professional men in the federal service. Many of these men have at some time been instructors in a university, in addition, many have had experience in industry. In their daily work they are faced with the problems of the practice of their professions, thus they carry to the classroom healthy enthusiasm and vigor, and a balance between theory and administration that is rarely duplicated.

The sponsors of the Graduate School have in mind a function of importance outside the scope of the great universities of the country. The School gives no degrees. Rather, the aim is to supplement the education of men and women in the federal service who for the most part already have degrees but who now, in their maturity, foresee the need of other training. Classes are held after official hours, not to interfere with official duties.

Among the professions represented in Washington probably none rates higher than the statisticians', and perhaps no profession was ever so concentrated in one city. During the past few years there has been increasing recognition and dependence among other professional groups and management in the service of both government and industry on current quantitative information which the trained statistician alone is capable of obtaining and evaluating. In the economic programs that are at present under consideration there will be increased demand for samples that will provide reliable national and local totals and indexes of agricultural production, population shifts, consumer preference, employment, unemployment, rent, sales, inventories (as of shoes and tires), vacancy and occupancy, opinions, and the like, at low-cost and quick timing. These requirements call for a mixture of the highest grade of experience and research in sampling theory, with administrative ability and knowledge of various lines of subject-matter.

Professor Hotelling, in his article in the April issue of the *Biometrics Bulletin*, pointed

out the need for increased specialization in statistical work and its teaching. His contention must be supported and his recommendations put into action; otherwise the disparity between the supply and demand for competent statisticians will widen to the point where the rest of the world will learn to thrive without statistics, or on a mediocre product. The need of specialization is made imperative by the increasingly wide variety of projects which statisticians must be prepared to work in, owing largely to the splendid statistical work that has recently been done in the War Department, Navy, N. D. R. C., industry and government civilian agencies. The statistician will henceforth find himself increasingly indispensable in all phases of industry and government, from the purchase of materials, studies of consumer preferences, consumer standards, design of product, control of quality in manufacturing, purchasing, and selling; also in personnel administration, polling, and social and economic surveys of all kinds so necessary in the successful administration of government or business. A statistician in the government service must be prepared to work on the design of samples for population or agricultural characteristics, inventories, rent, vacancy, number of stores, expenditures, number of workers, cost of handling furs or manufacturing shoes, or a particular price line of coats. Statisticians in industry and Agricultural Experiment Stations could likely match this list with an equally varied one, but the point is that the statistician must be a specialist in his own right, and be transferable over a wide variety of applications. In the Graduate School the Department of Statistics has always been separate and has never been overshadowed by any other department. The mathematics courses there are primarily directed toward providing prerequisites for more advanced work in statistics, with understandable exceptions as where a group has asked for a course in hydrodynamics or thermodynamics.

In the study of statistics in the Graduate School, emphasis is placed on the needs of the federal service. The word "efficiency"

in the design of samples for government uses refers to the amount of information (as measured by the coefficient of variation) furnished per unit cost, not per case. Moreover, in computing costs it is necessary to take into consideration not only the field and office costs of collecting and tabulating the data but the cost represented as the burden of response on the individuals or firms that furnish the information. There have been many instances in which a decision was made to take a sample less than 100 percent when the cost of so doing, to the government, was obviously going to be equal to or greater than the cost of complete count, the motive being to lessen the burden of response on industry.

The point of view in building the statistical curriculum is that while thorough training in mathematical statistics is the prime requisite of a statistician, he is nevertheless expected to perform in other capacities as well. For instance, he must know a great deal about questionnaires, and field and office instructions, and must know the subject-matter of the enquiry well enough to say what data will answer which questions, and what data can be collected. He must know whether to use a complete count or a sample short of a complete count or some other type of partial investigation, for all of which he must have pretty clear ideas on the cost and time required to carry out jobs of various complexities. He must look ahead and foresee the need of data, thus to recommend statistical programs ahead of time and not get caught short without data that should have been collected last quarter. Equally, he is expected to be strong in the analysis of good data and bad, and should be ready to translate his analyses into recommendations for action, tersely and forcefully, in simple language for administrative use (a rare ability indeed). The instructors face these problems daily in their work and do not find it difficult to stress them. Some of these special problems call for special courses, an example being Likert and Cannel's "Techniques of Interviewing," and Jaffe's "Planning of Statistical Enquiries."

The sampling courses offered in the Graduate School represent at present about the only opportunities for learning the new devel-

opments in areal and cluster sampling, and their impact on business surveys, much of which has not yet been published. Some of the sampling courses apparently have almost identical titles and content, but are given by different instructors. The aim is to provide the opportunity of getting acquainted with the Washington specialists and their contributions.

The catalog of the School (obtainable on request to the Director or the Chairman named below) lists a wide variety of courses in many phases of statistical technology and administration. The curriculum undergoes continual revision with the aim of providing courses that will supply needs of the future.

Special stress is placed on the courses in statistics of under-graduate caliber, wherein the students are largely people who have had considerable training in economics, sociology, agricultural science, or some other field. The maturity of the classes calls for instruction of the highest leadership. In the graduate levels most of the students are engaged in statistical work of some kind in various fields designing samples, working on interviewing and office instructions, questionnaires, analyses and adjustments, and writing reports and recommendations.

The Seminars in "Sampling" and "Statistical Inference" are held approximately every three weeks and are open to all graduate students and to others upon application. They are attended by most of the leading mathematically inclined statisticians and economists in Washington, the average attendance being about 60. Statisticians from outside of Washington often appear on the programs. It was the practice before the war to hold special series of lectures by outside speakers: Fisher, Shewhart, Neyman, Wishart, Yates and Cochran have appeared on these programs.

The Department Committee on Mathematics and Statistics is composed of O. C. Stine (Bureau of Agricultural Economics), P. M. Hauser (Census), B. Ralph Stauber (War Relocation Authority), M. A. Grishick (Bureau of Agricultural Economics and Columbia University), with W. Edwards Deming (Bureau of the Budget) as Chairman.



## NEWS AND NOTES

In a recent letter, MARIANNE E. BERNSTEIN wrote: "When reading the very interesting article by Dr. Phillip Levine on human blood groups in the current issue of the Biometrics Bulletin I noticed that he refers to my father Dr. FELIX BERNSTEIN as 'A mathematician who never carried out any blood tests'. I was a little girl with long pigtailed when my father was doing the research on the AB blood groups, and both my brother and I had our ears pricked by our father; and he then examined our blood in a laboratory installed in the Institute of Mathematical Statistics at the University of Gottingen. Incidentally, the entire family turned out to have blood of the 'O' type."

A copy of the above letter was sent to Dr. PHILLIP LEVINE and a portion of his reply to "This very delightful controversy" follows: "The late Dr. Landsteiner, with whom I was associated from 1925-1932, frequently mentioned that Dr. Bernstein, an outstanding mathematician, was able to arrive at the accepted theory of the heredity of the four blood groups without doing any blood group determinations. We had silently assumed that this was the case. If I am in error, I hasten to apologize, both to Dr. Bernstein and his daughter. It seems to me however, that essentially, it was not necessary for Dr. Bernstein to do any blood tests himself. It is a great tribute to him that direct application of his special gifts to a series of figures—representing the varying incidence of the four blood groups among several races studied by other workers, particularly Hirschfeld—resulted in his presentation of the triple allelomorph theory of the heredity of the four blood groups."

A Nutrition Work Conference, the third in a series of conferences being conducted for Southern research investigators in various fields, was held in Raleigh, May 14-19, sponsored by the General Education Board and the Institute of Statistics. The program included intensive training in statistics along with discussions and consultations on individual research problems. R. E. COMSTOCK, GERTRUDE M. COX, J. A. RICNEY, W. A. HENDRICKS, SARAH PORTER and JEANNE FREEMAN, members of the staff of the Institute of Statistics, gave lectures and led discussions on such topics as the uses of statistics in experimentation, available experimental designs and their applicability to nutrition research, summarizing data, the proper error term, and estimation of work necessary to obtain results with a specified degree of accuracy. In attendance were GEORGIAN ADAMS, U.S.D.A., Office of Experiment Stations; RUTH BOYDEN, University of Kentucky; ROBERT CAROLUS, Virginia Truck Experiment Station; MARY DODDS, University

of Tennessee; J. F. EHEART, Virginia Agricultural Experiment Station; PETER HEINZE, Vegetable Breeding Laboratory at Charleston; MARTHA HOLLINGER, Louisiana State University; HAROLD M. HYRE, West Virginia University; BYRON E. JAMES, University of Florida; SOPHIE MARCUSE, Bureau of Nutrition and OLIVE SHEETS, Mississippi State College; Home Economics; LAVERNE MCWHIRTER, Mississippi State College; RUSSEL MILLER, Pennsylvania State College; W. J. PETERSON, North Carolina State College; RUTH REDER, Oklahoma Agricultural Experiment Station; F. W. SHERWOOD, North Carolina State College; OLIVE SHEETS, Mississippi State College; MARY SPEIRS, Georgia Agricultural Experiment Station; and JESSIE WHITACRE, Texas Agricultural Experiment Station.

JOHN WISHART wants to clear up his whereabouts which was incorrectly reported in the April Bulletin . . . He is with the Admiralty, heading an administrative statistical group.

COLONEL JOSEPH BERKSON, on leave of absence from the Mayo Clinic, has been in the Army since May 1942 in his present position as Chief of the Statistics Division of the Office of the Air Surgeon. He says that he is supposed to be responsible for anything that pertains to medical or biological statistics in the Army Air Forces. That birthday in May put him over the draft limit so maybe we can get him back into civilian life and on the editorial committee of the Bulletin . . . ROBERT P. GAGE is carrying on in the Department of Biometry and Medical Statistics at the Mayo Clinic. His son Billy and daughter Roberta are keeping him busy at home since help is at a premium. He says, "We are living at a fast pace" . . . Lt. J. A. GREENWOOD, formerly of Duke University, is now stationed at the U. S. Naval Air Station, Patuxent River, Md., the testing station for the Bureau of Aeronautics. He is statistician at the armament test unit, and states, "I am having fine co-operation in the introduction of statistical design and evaluation in their tests." Wonder if his experience with extra-sensory perception helps with the armament tests . . . ALAN E. TRELOAR was found at Fort Preble, Maine, working on the wet-cold field trials", but he is back at the Climatic Research Laboratory, Lawrence, Massachusetts, formulating reports. He hopes to return to the University of Minnesota about June 15. If you succeed let us know how one gets a release from the Statistical Research Group . . . Perhaps, after June 17, a change in age will bring a change in heart to S. S. WILKS and then maybe he will release R. L. ANDERSON, who is needed back on his job at North Carolina State College . . . CAPTAIN CARL E. MARSHALL of the Department of Mathe-

matics, Oklahoma A & M College is at the Advanced Bombardier School, Big Spring, Texas . . . Have you noticed the letter heading "Columbia University, Division of War Research, Statistical Research Group" with HAROLD HOTELLING as official investigator and W. ALLEN WALLIS as director of research? We have been promised a fairly full statement about this group after the war . . . ABRAHAM WALD has been promoted to a professorship of mathematical statistics at Columbia University . . . P. L. HSU of Kunming, China, will lecture on advanced multivariate analysis and on other fields in mathematical statistics at Columbia during the semester beginning on January 31, 1946 . . . JOSE CALZADA, agronomist and statistician from the experiment station at Lima, Peru, is studying statistics at Iowa State College. He is one of the Rockefeller Foundation Fellows and expects to travel extensively this summer visiting experiment stations in the United States. W. G. COCHRAN, Civilian U.S.S.B.S., APO 413 %Postmaster, New York, writes, "London distressed me a little at first, but one soon gets used to the missing houses, the substitution of boards for windows, and the general air of dilapidation. The vivid green in the fields and parks is a heartening sight." He is working long hours and keeping the information in his head since there is no paper available. He witnessed the impressive two-day V-E celebration in London with the lights ablaze, the people singing, and the flags waving. More recently he wrote from Germany that he is getting somewhat inured to the constant scenes of destruction. He and his group are billeted in tastefully furnished homes of former Nazi officials. The German people are making their trek back to the demolished towns. He wrote that the hull of the Cologne Cathedral was standing, and the two spires were still an impressive landmark, but there had been considerable gutting inside which would require long and careful reconditioning. How about celebrating July 15 in the United States? . . . SAMUEL SIMONOVITZ has been transferred from the Office of Milk Administrator to the State Development Commission in Hartford, Connecticut, where he will be concerned with the correlation of information

from state and federal agencies to provide a statistical basis for the administrative policies of the Commission . . . LOWELL J. REED at Johns Hopkins University finds much of his time taken up with war work in addition to his ordinary duties. We can hear others saying "me too!" . . . E. R. PARKER, Citrus Experiment Station, Riverside, Calif., says, "I have had to shelve, I hope temporarily, the work on the incorporation of regression in the analysis of variance of yields of long-term experiment. A very urgent problem concerning the 'quick decline' of orange trees is being undertaken. Similar troubles with sour orange rooting trees have been observed in South Africa, South America and Java. I am attempting to put in force this spring a 5 x 5 Latin square experiment on effects of cultivation in citrus orchards." His border rows of grapefruit trees produce excellent fruits! . . . G. A. BAKER, Division of Mathematics and Physics, says they have a large group of men there at Davis, Calif., who have studied statistics to a certain extent and who use it constantly in their research work . . . H. L. LUCAS, research associate, Department of Poultry Husbandry at Cornell University, has been conducting an informal class on experimental designs, analysis of data and interpretation of results . . . L. N. HAZEL with the Bureau of Animal Industry located at Dubois, Idaho, sent in some good suggestions about this bulletin. *Tell us what you want included in future issues* . . . O. A. POPE with the Plant Industry Section at Beltsville writes that he feels the section on Queries should be especially useful as a medium for obtaining answers on questions which frequently arise in the course of research work. Send G. W. SNEDECOR your questions. So far, he leads with "fan mail" . . . WALTER H. MEYER has recently been promoted to Professor of Forestry at the Yale University School of Forestry, where he gives courses in statistical methods, forest management, forest economics and related fields. Professor Meyer is President of the Connecticut Chapter of the American Statistical Association . . . The chairman of the Editorial Committee refused to accept reports on the recent trip of GERTRUDE COX to New York City and New Haven, Conn.

## QUERIES

**QUERY** A problem that has bothered me is the fitting of regression lines when their position is restricted in some way. For example, suppose a test is made of the relationship between the number of fish in a body of water and the average number which can be caught out of it, with a standard amount of fishing. In fitting a regression line to such data, we know that the point (0,0) must fall on the line, since if no fish are present certainly none

will be caught. In other words, we have one point which is free from sampling error. The unique importance of this point will, it seems to me, make observations in its neighborhood of relatively less importance than observations at a distance from it, where there is no fixed guide-post. Do you know of any treatment of situations of this sort, by which the best straight (or curved) line could be fitted to data when there is one point which *must* be



satisfied? The standard deviation from regression ("standard error of estimate") and the standard error of the regression coefficient, would also be valuable. Or are these concepts pertinent in such a situation?

ANSWER Deming gives both a general method and some particular solutions of your problem (Statistical Adjustment of Data. John Wiley and Sons, Inc., pages 30-34). Snedecor opens his chapter 6 with an illustration of the simple case in which X is measured without error and the variance of Y is constant for all values of X (Statistical Methods. Collegiate Press, Inc., Ames, Iowa).

Observations in the neighborhood of (0,0) may or may not be of less importance than those at greater distances; it depends upon the variance of Y. One often finds that this variance increases with X. In fact, there are many situations in which it seems reasonable

to suppose that in the sampled population the standard deviation of Y is directly proportional to X. If you think this hypothesis is suitable in your fishing, the appropriate method is to calculate the ratios,

$$\frac{X}{Y} = \frac{\text{number of fish caught}}{\text{total number of fish}}$$

then apply to them the statistical procedures suitable for a single variate.

GEORGE W. SNEDECOR.

QUERY The data in the table were analyzed by the method of fitting constants, the (incomplete) results being shown at the end. Are we justified in using this or any other statistical approach on the whole body of data when the amount of missing information is as great as it is here? What are the degrees of freedom for interaction?

TABLE 1. CAROTENE IN BLOOD PLASMA OF CATTLE.  
Micrograms per 100 ml. blood plasma (exalted).

Month of observation	No. animals and mean		Age of animals (months)			
		1-3	4-6	7-9	10-12	
August	n	5	8	6		
	$\bar{X}$	206.6	379.1	517.2		
June	n	8	6	5		
	$\bar{X}$	192.1	478.0	456.6		
April	n	5	6			
	$\bar{X}$	93.6	205.2			
March	n	9	2			
	$\bar{X}$	105.6	189.5			
February	n	7				
	$\bar{X}$	120.3				
December	n	4				
	$\bar{X}$	61.0				
October	n	1	1	3	7	
	$\bar{X}$	55.0	233.0	289.7	282.4	
September	n	2	2	6	6	
	$\bar{X}$	75.0	263.5	254.0	231.2	

*Analysis of variance*

Source of variation	Degrees of freedom	Sum of squares	Mean square
Months	7	569,737	81,398
Ages	3	660,602	220,201
Interaction	?	104,541	
Individuals within subclasses	79	464,994	5,886

ANSWER Taking the second question first: the degrees of freedom for interaction are

(7) (3)—number of vacant subclasses; that is,  $21-12=9$ . Hence, the mean square for interaction is  $104,541/9=11,616$  and  $F$  is  $11,616/5,886=1.97$ , just at the 5% point.

In the method which you have used for fitting constants the hypothesis is set up that in the sampled population the interaction is negligible (F. Yates. *Jour. Agri. Sci.* 23:108; and *Emp. Jour. Exp. Agri.* 1:129). The test of significance indicates rejection of that hypothesis. But examination of the data reveals rather orderly progressions of the means with increasing age except in August when the mean for 7-9 went on up instead of following the pattern. Since there are 6 cattle in this subclass, it may account for a considerable portion of the interaction. This should be considered before rejecting the hypothesis of negligible interaction. If it should be found that these 6 animals were treated differently from others of the same age (kept longer on pasturage, for example), decision about the hypothesis might be affected. On the other hand, even if there is some interaction in the population, your estimates of mean square will not be greatly in error, and your decisions about the effects of month and age will not be changed.

Returning to your first question: Of course, you do not have the information that might be expected in a complete  $4 \times 8$  table, but the method used enables you to extract that which is available. I do not know the problem which the experiment was designed to solve, hence cannot judge the appropriateness of the method used. I always feel that information may be lost by classifying a continuous variate like age in such broad intervals as yours. If you are interested in the trend of the carotene content, covariance might be more suitable.

QUERY In your Queries Section of *Biometrics Bulletin* Vol. 1, No. 1, p. 9 you say "... the

null hypothesis states that the real relation between  $x$  and  $y$  is a straight line." Is that an acceptable null hypothesis? Shouldn't the null hypothesis be stated in negative terms? For example: "the real solution between  $x$  and  $y$  is not a straight line."

Is it possible to give a brief clear exposition of the advantages that were gained by the introduction of the term "null hypothesis" into statistical literature?

ANSWER In certain important tests of hypothesis, e. g., the  $t$ -test for the difference between two means, the hypothesis is often set up with the possibility of rejecting it at some designated significance level, and for this reason it is often called a "null hypothesis." In such cases the "null" part of the phrase "null hypothesis" refers to our interest in a possible rejection rather than that the null hypothesis should be stated negatively.

The phrase "null hypothesis" has come to be used by some statisticians in a much broader sense than the above. It has been used to designate any eligible hypothesis. In this sense the "null hypothesis," in order to be an eligible one, must be free from any vagueness in order to form the basis for the sampling distribution to be used in making tests of significance or in setting up confidence limits. The hypothesis "that the real relation between  $x$  and  $y$  is a straight line" is an eligible hypothesis in the sense that it is free from vagueness. It is, however, an hypothesis for testing a specification rather than an ordinary hypothesis in which the specification is assumed and tests are made of certain parameters estimated from the data. Such hypotheses may present difficult mathematical problems.

T. A. BANCROFT.

*Editor's note:* It is doubtless possible to give a clear exposition of the testing of hypotheses, but it may not be brief. This part of the question will be put on the docket for an expository article.



## ABSTRACTS

(11)

BERKSON, Joseph, M.D. (Mayo Clinic). Application of Logistic Function to Bio-Assay. *Jour. Amer. Stat. Assn.* 39:357-365 Sept. 1944

When the effect of a drug is measured as the proportion of exposed individuals affected, the effect plotted against dosage (or its logarithm) frequently assumes the form of a symmetric sigmoidal curve. From this curve the L.D. 50, i.e., the dose which affects just 50 per cent of those exposed, or any similar measure of dosage can be estimated. The integral of the normal curve has been used to represent the sigmoidal function, and its application advanced extensively by Bliss, with a method of linear transformation as "probits." This article advances the use of the "logistic function" and a linear transformation with the use of "logits." The curves are similar, but the logistic is analytically simpler in a number of respects. The "logit" is simply the logarithm of the ratio of  $p$  to  $q$ , where  $q$  is the proportion affected and  $p$  its complement; the first derivative which is utilized to determine the weights to be used in the solution is proportional to the simple product of  $p$  and  $q$ . Thus no auxiliary tables other than one for logarithms are required.

A method of fitting equivalent to that used for "probits" can be applied to "logits." In this method an approximation is implicitly used

$$x^2 \rightarrow \frac{\hat{Z}^2}{\hat{P}\hat{Q}} (l - \hat{L})^2,$$

where  $\hat{Z}$  is the first derivative with respect to the linear transformation of the fitted function,  $\hat{P}\hat{Q}$  the fitted rates and  $l, \hat{L}$  the linearly transformed variates of the observed and fitted rates respectively. For "probits"  $\hat{Z}$  is the ordinate of the normal curve; for "logits"  $\hat{Z}$  is equal to  $\hat{P}\hat{Q}$ , so for the logistic the approximation is simplified to

$$x^2 \rightarrow \hat{P}\hat{Q} (l - \hat{L})^2.$$

If "corrected" values are to be used to substitute for observed rates, the corrected logit is given by

$$e' = \hat{L} - \frac{(q - \hat{Q})}{\hat{P}\hat{Q}}.$$

This method, both in the case of "probits" and "logits," requires one or more preliminary solutions to obtain the values of  $\hat{P}\hat{Q}$  and  $\hat{Z}$ .

A different method is advanced in this article for the logistic, depending on a more precise approximation

$$x^2 \rightarrow \frac{z\hat{Z}}{\hat{P}\hat{Q}} (l - \hat{L})^2$$

where  $z$  corresponds to the observed and  $\hat{Z}$  to the fitted rate. For the logistic this simplifies to

$$x^2 \rightarrow pq (l - \hat{L})^2.$$

The weights now involve only the observed rates, and a final solution can be obtained at once, without successive approximation. This moreover should yield, for any given set of differences, a lower  $\chi^2$ . A trial for a number of previously published series gave a lower  $\chi^2$  in these instances for "logits" than for "probits."

(12)

GOODWIN, Richard H. (Connecticut College). Estimates of the Minimum Numbers of Genes *Solidago*. *Bulletin of the Torrey Botanical Club*.

Interspecific and intraspecific crosses in *Solidago* have been made between various strains. From analyses of the  $F_1$  and  $F_2$  populations, estimates have been made of the minimal numbers of genes differentiating these strains. Minimal estimates for single character differences were calculated from the following expression derived by Professor Sewall Wright:  $n = \Delta_x^2 / 8(R_x^2 - 1)$ , where  $n$  equals the number of gene differences,  $\Delta_x$  equals the difference between parental strains in average value of character  $x$  divided by the standard deviation of that character in the  $F_1$ , and  $R_x$  equals the standard deviation of character  $x$  in the  $F_2$  divided by the standard deviation of character  $x$  in the  $F_1$ . The maximal number of genes affecting the expression of any two characters,  $n_c$ , were obtained from the following formula derived by Dr. Donald R. Charles:

$$n_L(1 - 2c) + n_c = \Delta_y(r_2 R_x R_y - r_1) / 8(R_x^2 - 1)(R_y^2 - 1),$$

where  $n_L$  equals the number of character- $x$  loci each linked with a character- $y$  locus,  $c$  equals the average crossover value between linked pairs of genes,  $\Delta_y$  and  $R_y$  equal character- $y$  quotients analogous to character- $x$  quotients, and  $r_1$  and  $r_2$  equal the correlation coefficients between characters  $x$  and  $y$  in the  $F_1$  and  $F_2$  respectively. In making minimal estimates for the total number of genes differentiating all the characters analyzed,  $n_c$  was taken as the nearest integer below the exact value for the expression  $n_L(1 - 2c) + n_c$ .

Taxonomic distinctions in *Solidago* have been based upon morphological criteria. The number of genes involved in the control of some of the morphological differences in an interspecific cross has been estimated to be at least twenty-one; in an intraspecific cross, between strains which have been considered subspecifically distinct, at least four.

For practical reasons the systematist has taken little cognizance of physiological behavior in delimiting taxonomic categories. Yet in the two intraspecific crosses studied, at least nine or ten genes appear to be involved in differentiating the parental strains with respect to flowering time. In one case, the strains were morphologically dissimilar and subspecifically distinct; in the other case, morphological differences were inadequate to warrant a taxonomic separation.

KNUDSEN, Lila F. (Food and Drug Administration). *Penicillin Assay. Sci. 101:46 Jan. 12, 1945.*

A simple statistical method is given for determining the potency of antibiotic substances, in terms of a suitable standard and error of assay, by means of a chart and a nomograph in conjunction with four numbers obtained by certain additions and subtractions of the diameters of the zones of inhibition of the incubated plates. Each plate has a low and a high dose of standard having diameters of zone of inhibition  $s_L$  and  $s_H$  respectively, and a low and high dose of the unknown having diameters  $u_L$  and  $u_H$ . For each plate two quantities are calculated:

$$v = (u_L + u_H) - (s_L + s_H)$$

and

$$w = (s_H + u_H) - (s_L - u_L).$$

The four quantities used in conjunction with the chart and nomograph are  $V = \Sigma v$ ,  $W = \Sigma w$ ,  $R_v$  = range of  $v$  and  $R_w$  = range of  $w$ . No further calculations are necessary.

Although they apply specifically to a four-plate assay in which the ratio of high dose to low dose is 4:1, the chart and nomograph can be used for any number of plates and any ratio of doses by multiplying the result obtained by an appropriate tabled figure.

The method can be used also in similarly designed assays of other drugs or vitamins where, for instance, differences between litters are to be removed.

KNUDSEN, Lila F. (Food and Drug Administration). *Control Chart Analysis of Penicillin Assays. Jour. of Bact. Aug. 1945.*

Control Chart Analysis can be applied to the penicillin assay abstracted above by setting up control charts for  $W$ ,  $R_w$  and  $R_v$ .  $W$  is a constant function of the slope of the dosage response curve.  $R_w$  is a measure of the variation of these slopes from plate to plate and  $R_v$  is a measure of the between-plate variation of the vertical distance between the two parallel dosage response curves fitted to the data from each plate. Averages and three-sigma control limits on  $W$ ,  $R_w$  and  $R_v$  are calculated from data gathered at one laboratory over a few weeks. A separate control chart is made for each of the three variables with assay number (or time) as the abscissa and the variable ( $W$ ,  $R_w$ , or  $R_v$  as the case may be) as the ordinate. The averages and control limits are plotted on the charts. As each assay is run the values of  $W$ ,  $R_w$  and  $R_v$  are plotted. If the plotted point is outside the control limits, trouble is indicated, possibly in the form of contamination or a leaky cup. After the trouble is located the assay is repeated.

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